MISSOURI WESTERN STATE UNIVERSITY

SCHOOL OF LIBERAL ARTS AND SCIENCES

DEPARTMENT OF COMPUTER SCIENCE, MATHEMATICS, AND PHYSICS

COURSE NUMBER: MAT 462

COURSE NAME: Number Theory

COURSE DESCRIPTION:

A 3 semester hour course studying divisibility, primes, congruences, Diophantine equations, arithmetic functions, partitions, Fibonacci numbers, and continued fractions.

PREREQUISITE:

MAT 208

TEXT:

COURSE OBJECTIVES:

The goal of this course is to introduce the student to the field of number theory. In order to meet this goal, the students will learn how to:

1. Use the "divides" relation in proofs of properties involving *a/b*.
2. Prove some properties of prime numbers.
3. Apply the "well-ordered principle" in proofs.
4. Understand and apply the prime factorization theorem.
5. Understand and apply the division algorithm.
6. Use the greatest integer function.
7. Work with the L.C.M. and the set Ma + Mb.
8. Find set product and set sum.
9. Work with the G.C.D. and the set Ma. + Mb.
10. Determine if numbers are relatively prime.
11. Understand and apply the Euclidean algorithm.
12. Use "Diophantine equations" in number theory.
13. Use the divisors of a product in number theory.
14. State and use the fundamental theorem of arithmetic.
15. Work with continued fractions.
16. Work with residue classes.

17. Prove theorems involving congruence.
18. Do arithmetic of congruence.
19. Work with arithmetic functions.

STUDENT COMPETENCIES:

1. Prove statements involving sets and subsets of divisors.
2. Find $\nu(n)$, $\tau(n)$ and $\sigma(n)$ for a given n.
3. Prove theorems using the well-ordering principle.
4. Work with the division algorithm.
5. Find the set product and the set sum.
6. Use the greatest integer function.
7. Find L.C.M. and G.C.D.
8. Find the prime factorization of a positive integer.
9. Prove theorems involving relatively prime.
10. Use the Euclidean algorithm in working problems.
11. Learn the fundamental theorem of arithmetic.
12. Learn the meaning of residue classes.
13. Know characterizations of $a = b \pmod{n}$.

COURSE OUTLINE:

1. The Integers
   - A. The Set of Integers Relations "divides"
   - B. Well Ordering
   - C. Other Methods of Proof

2. Divisibility
   - A. The Relation "Divides"
   - B. Least Common Multiple and Greatest Common Divisor
   - C. Properties of the GCD and Euclidean Algorithm
   - D. The GCD as a Linear Form $ax + by$
   - E. Linear Diophantine Equations

3. Continued Fractions
   - A. Continued Fraction Representations of Rational Numbers
   - B. Continued Fraction Representations of Irrational Numbers
   - C. Convergents

4. Prime Numbers and the Fundamental Theorem of Arithmetic
   - A. The Primes
   - B. Special Primes and Unsolved Problems

5. Congruences
    A. Congruence
    B. Divisibility Tests
    C. Residue Classes and Modular Arithmetic
    D. Solving Linear Congruences
    E. The Chinese Remainder Theorem

6. Arithmetic Functions
    A. The Tau Function
    B. The Sigma Function
    C. The Phi Function