MISSOURI WESTERN STATE UNIVERSITY

COLLEGE OF LIBERAL ARTS AND SCIENCES

DEPARTMENT OF COMPUTER SCIENCE, MATHEMATICS, AND PHYSICS

COURSE NUMBER: MAT 416

COURSE NAME: Abstract Algebra

COURSE DESCRIPTION:

Groups, rings and fields; definitions and fundamental theorems; homomorphisms and isomorphisms; and polynomials and field extensions. The topics included in MAT 416, and the emphases placed on those topics, are influenced by the needs of prospective secondary school mathematics teachers. 3 credit hours. Spring.

PREREQUISITE:

Grade of C or better in MAT 306 or equivalent

TEXT:

Elements of Modern Algebra, Gilbert and Gilbert, Brooks, Edition 6th 05, Cengage L, ISBN 9780534402648

COURSE GOALS:

Three major goals of this course are:

1. To provide students with knowledge and understanding of algebra as one of the major branches within the discipline of mathematics.

2. To develop understanding and appreciation of the nature and purpose of axiomatic systems.

3. To provide prospective teachers of second school mathematics with a broad perspective of several subjects taught at the secondary school level.

COURSE OBJECTIVES:

In order to meet these major goals, students will:

1. Demonstrate understanding of the concepts of relation, function, composition, and inverses.

2. Extend the system of integers to the system of rationals, the system of rationals to reals, and the reals to the complex number system.

3. Use the defining postulates, definitions, and theorems to establish properties of the various number systems.

4. Prove theorems relating to groups, subgroups, cyclic groups, and permutation groups.

5. Show that seemingly different mathematical structures may be essentially the same (isomorphic).

6. Identify by name and algebraic properties groups, rings, integral domains, and fields.

7. Compare and contrast the domain of integers and the domain of polynomials.

6. Complete a notebook containing solutions to assigned problems and proofs.

7. Discuss solutions and proofs with other members of the class using appropriate mathematical language and symbolism.

10. Develop an understanding of various number systems including the integers, rationals, reals, and complex numbers, and algebraic techniques in these systems.

STUDENT COMPETENCIES:

In order to meet the above objectives, successful students will:

1. Construct operation tables for selected binary operations.

2. Determine if a given binary operation is commutative, associative, or if the identity and inverses exist.

3. Prove or disprove whether a given relation is an equivalence relation.

4. Utilize mathematical induction to establish properties of the integers.

5. Prove theorems related to divisibility in the system of integers.

6. Solve congruence modulo n.

7. List the properties of a group and Abelian group.

8. Prove basic theorems relating to groups, subgroups, cyclic groups, and permutation groups.

9. Prove that two groups are isomorphic.

10. List the defining properties of a ring, ring with unity, and commutative ring.

11. Prove that a given set is a ring.

12. List the properties of an integral domain.

13. Prove that a given set is an integral domain.

14. Construct the field of quotients of an integral domain.

15. Define an ordered integral domain.

16. Prove theorems involving the order relations.

17. Define ring homomorphism.

18. Prove theorems relating to ring homomorphisms.

19. Define a complete ordered field.

20. Prove existence of irrational numbers.

21. Extend the field of real numbers to the complex number system.

22. Prove DeMoivre's Theorem.

23. Find the n n^{th} roots of a complex number.

24. Define the set of polynomials over an integral domain.

25. Prove theorems related to polynomials.

COURSE OUTLINE:

1. Sets, Mappings, and Binary Operations

    1. Operations on Sets
    2. Relations and Functions
    3. Composition of Functions
    4. Binary Operations

2. The Integers

    1. Mathematical Induction
    2. Divisibility
    3. Prime Factors and Greatest Common Divisor
    4. Congruence of Integers
    5. Congruence Classes

3. Groups

    1. Definition of a Group
    2. Subgroups
    3. Cyclic Groups
    4. Permutation Groups
    5. Isomorphisms
    6. Finite Permutation Groups
    7. Normal Subgroups
    8. Quotient Groups and Homomorphisms

4. Rings, Integral Domains, and Fields

    1. Definition of a Ring
    2. Integral Domains and Fields
    3. The Field of Quotients of an Integral Domain
    4. Ordered Integral Domains
    5. Ring Homomorphisms
    6. The Characteristic of a Ring

5. Real and Complex Numbers

    1. The Field of Real Numbers
    2. The Complex Numbers
    3. DeMoivre's Theorem and Roots of Complex Numbers

6. Polynomials

1. Polynomials Over a Ring
2. Divisibility and the Greatest Common Divisor
3. Factorization in F[x]
4. Zeros of a Polynomial