MISSOURI WESTERN STATE UNIVERSITY

COLLEGE OF LIBERAL ARTS AND SCIENCES

DEPARTMENT OF COMPUTER SCIENCE, MATHEMATICS, AND PHYSICS

COURSE NUMBER: MAT 407

COURSE NAME: Advanced Calculus I

COURSE DESCRIPTION:

Elementary topological aspects of the real numbers, sequences, limits and continuity, differentiation, integration, and infinite series.

PREREQUISITE:

Grade of C or better in MAT 306.

TEXT:

Understanding Analysis, Abbot

COURSE OBJECTIVES:

The major goal of this course is to provide students with understanding and proficiency sufficient to enable successful students to pursue further study in advanced mathematics and in disciplines which rely upon understanding of or proficiency in advanced calculus. Students will be exposed to a theoretical development of the calculus concepts that are presented intuitively in the typical calculus course. Students will be expected to write formal proofs. In order to meet this major goal, students will learn how to:

1. Utilize definitions and assumptions to prove properties of the real numbers.
   (MoStep Mathematics Competencies 6.1)

2. Prove basic properties of sequences using definitions and appropriate theorems.
   (MoStep Mathematics Competencies 6.1, 8.3)

3. Understand and prove basic theorems of the real numbers such as: Bolzano-Weierstrass Theorem, Theorems on Cauchy Sequences, the Nested Interval Theorem, and the Heine-Borel Covering Theorem.
   (MoStep Mathematics Competencies 6.1)

4. Develop the theory of continuous functions.
(MoStep Mathematics Competencies 8.1)

5. Understand the consequences of continuity such as the Intermediate Value Theorem.
(MoStep Mathematics Competencies 8.1)

6. Develop the theory of the derivative.
(MoStep Mathematics Competencies 8.1)

7. Develop the theory of the Riemann Integral and the Riemann-Stieltjes Integral.
(*MoStep Mathematics Competencies 8.1)*

8. Derive tests for infinite series and power series.
(MoStep Mathematics Competencies 8.3)

9. Solve problems which require application of concepts of limit, continuity, differentiation, integration, and other continuous processes.
(MoStep Mathematics Competencies 1.1, 8.1)

10. Utilize mathematical language and symbolisms to communicate ideas and represent relationships.
(MoStep Mathematics Competencies 1.2)

11. Interact verbally with others to clarify and extend understanding of mathematical situations.
(MoStep Mathematics Competencies 1.2)

STUDENT COMPETENCIES:

In order to meet the above objectives, successful students will:

1. Understand the concept of function as a unifying idea in mathematics.

2. Develop basic properties of the real numbers utilizing the Lease Upper Bound Axiom.

3. Prove basic properties of sequences using definitions and properties of real numbers.

4. Be able to show functions are continuous by the Sequential Criterion for continuity.

5. Be able to use the Sequential Criterion for Function Limits.

6. Understand uniform continuity and the Sequential Criterion for uniform continuity.

7. Derive the Chain Rule for the derivative and apply it on composite functions.

8. Derive the Fundamental Theorem of Calculus and related theorems.

9. Evaluate different types of improper integrals.

10. Utilize various tests of convergence for infinite series.

11. Read, with understanding, written presentations of Mathematics.

12. Prove statements using the definition of the derivative.

13. Prove statements using the definition of the Riemann Integral.

14. Interact verbally with others to clarify and extend understanding of mathematical situations.

15. Use mathematical reasoning to:

    1. Follow logical arguments.
    2. Make and test conjectures.
    3. Judge validity of logical arguments.
    4. Formulate counter examples.

16. Construct formal proofs using direct and indirect arguments and mathematical induction.

17. Appreciate the power of the axiomatic approach in developing mathematical truth.

18. Understand analysis as one of the major branches of mathematics.

19. Demonstrate the interrelationship of the different branches of mathematics.

COURSE OUTLINE:

1. Mathematical Statements and Proofs

    1. Types of Mathematical Statements
    2. The Structure of Proofs

2. Ordering of the Real Numbers

    1. The Order Axiom
    2. Lease Upper Bounds
    3. The Density of the Rational Numbers

3. Sequence Limits

    1. Convergent Sequences
    2. Algebraic Combination of Sequences
    3. Infinite Limits
    4. Subsequences and Limit Points

5. Monotonic Sequences

4. Completeness of the Real Numbers

    1. The Bolzano-Weierstrass Theorem
    2. Cauchy Sequences
    3. The Nested Intervals Theorem
    4. The Heine-Borel Covering Theorem

5. Continuous Functions

    1. Continuity
    2. The Sequential Criterion for Continuity
    3. One-Sided Continuity
    4. Function Limits
    5. The Sequential Criterion for Function Limits
    6. Variations of Function Limits

6. Consequences of Continuity

    1. The Range of a Continuous Function
    2. The Intermediate Value Property
    3. Uniform Continuity
    4. The Sequential Criterion for Uniform Continuity

7. The Derivative

    1. Difference Quotients
    2. The Chain Rule
    3. The Law of the Mean
    4. Cauchy=s Law of the Mean
    5. Taylor=s Formula with Remainder
    6. L=Hopital=s Rule

8. The Riemann Integral

    1. Riemann Sums and Integrable Functions
    2. Basic Properties
    3. The Darboux Criterion for Integrability
    4. Integrability of Continuous Functions

5. Products of Indegrable Functions
6. The Fundamental Theorem of Calculus

9. Improper Integral

1. Types of Improper Integral
2. Integral Over Unbounded Domains
3. Integral of Unbounded Function
4. The Gamma Function
5. The LaPlace Transform

10. Infinite Series

1. Convergent and Divergent Series
2. Comparison Tests
3. The Cauchy Condensation Test
4. Elementary Tests
5. Absolute and Conditional Convergence
6. Regrouping and Rearranging Series
7. Multiplication of Series

11. The Riemann-Stieltjes Integral

1. Functions of Bounded Variation
2. The Total Variation Function
3. Riemann-Stieltjes Sums and Integral
4. Integration by Parts
5. Integrability of Continuous Functions