**LOGARITHMS**

What is a logarithm?

As with anything in mathematics, for one operation, there is an inverse operation for it. The logarithm function \( y = \log_a x \) is the inverse of the exponential function.

**Basic Elements of a Logarithm**

Let’s establish the basics of the logarithm function.

\[ \log_{10} 1000 \] is an example of a logarithmic expression

The lowered/subscripted number is the base of the logarithm. Many times the base is not written, and in such cases, 10 is the base for a logarithm. This is called the *common logarithm*.

\[ \log_{10} 1000 \] is an equivalent statement to \( \log 1000 \)

To properly refer to this expression, we say: “log base 10 of 1000”

The 1000 is actually the answer when raising the base to an exponent, as you see below:

To evaluate \( \log_{10} 1000 \), think: “What exponent do I raise the base 10 to, to get 1000?”

Or, “What exponent satisfies \( 10^x = 1000 \)?”

The answer is 3, so \( \log_{10} 1000 = 3 \).

**Note:** For \( \log_a x \), we do not use \( a = 1 \) as a base. Also \( a > 0 \) and \( x > 0 \)

**Changing Forms** (This skill will be used in solving logarithmic and exponential equations.)

In logarithmic form, \( \log_a b = x \) is equivalent to the exponential form \( a^x = b \).

![Diagram of logarithmic expression](image)
Properties of Logarithms

1. $\log_b(m \cdot n) = \log_b m + \log_b n$
   
   Example: $\log_{10}(5 \cdot x) = \log_{10} 5 + \log_{10} x = 0.698970004 + \log_{10} x$

2. $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
   
   Example: $\log_{10}\left(\frac{10}{x}\right) = \log_{10} 10 - \log_{10} x = 1 - \log_{10} x$

3. $\log_b(m^n) = n \cdot \log_b m$
   
   Example: $\log_{10} 10^2 = 2 \cdot \log_{10} 10 = 2 \cdot 1 = 2$

4. $\log_b m = \frac{\log_a m}{\log_a b}$
   
   Example: $\log_5 8 = \frac{\log_{10} 8}{\log_{10} 5}$ (Each with a base of ten. It can now be evaluated in a calculator.)

$= 1.292029674$

5. $\log_b(b^n) = n$
   
   Example: $\log_{10} 10^2 = 2$

6. $b^{\log_b m} = m$
   
   Example: $5^{\log_5 2} = 2$

7. If $\log_b M = \log_b N$, then $M = N$.
   
   Example: $\log_5(2x + 1) = \log_5 7$ so, $2x + 1 = 7$ and now solve for $x$.

8. $\log_b b = 1$

9. $\log_b 1 = 0$

Final Notes

- A logarithmic expression with base $e$ ($\log_e x$) is equivalent to $\ln x$; this is the natural logarithm.

Examples:

$log_{10} 100 = 2$

$ln 100 = 4.605170186$