Working with Circles

**Definition of a Circle:** A circle is the set of all points that are the same distance from a fixed point called the center. The distance from the center to any point on the circle is called the radius.

**Standard Form of a Circle:**
The standard form of the equation of a circle with center \((h, k)\) and radius \(r\) is:

\[(x - h)^2 + (y - k)^2 = r^2\]

**Example:** Let \((h, k) = (4, -7)\) and \(r = 3\). Write the equation of this circle in standard form. Plugging into the equation 4 for \(h\), -7 for \(k\), and 3 for \(r\) you get:

\[(x - 4)^2 + (y + 7)^2 = 9\]

**Completing the Square:**
The equation of a circle is not always given to you in standard form. So, you have to use the method of completing the square to rewrite the equation into standard form.

**Example:** You must complete the square for BOTH variables in the equation of a circle.

\[
x^2 - 4x + y^2 + 6y - 3 = 0
\]

\[
x^2 - 4x + y^2 + 6y = 3
\]

\[
(x^2 - 4x + \text{__}) + (y^2 + 6y + \text{__}) = 3
\]

\[
\left(\frac{4}{2}\right)^2 \quad \left(\frac{6}{2}\right)^2
\]

\[
(x^2 - 4x + 4) + (y^2 + 6y + 9) = 3 + 4 + 9
\]

\[(x - 2)^2 + (y + 3)^2 = 16
\]

**Finding the Center and Radius of a Circle:**
Find the center and radius of a circle that is written in standard form. Remember, the standard form of the equation of a circle is \((x - h)^2 + (y - k)^2 = r^2\) where \((h, k)\) is the center and \(r\) is the radius.

**Example:** \((x - 5)^2 + (y + 6)^2 = 49\)

**Center:** \((h, k) = (5, -6)\)  \quad **Radius:** \(r^2 = 49\), so \(r = \sqrt{49} = 7\)
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**Example:**  \( x^2 + (y - 9)^2 = 29 \)  
Think of it as \((x - 0)^2 + (y - 9)^2 = 29\)

**Center:** (0, 9)  **Radius:** \( r = \sqrt{29} \)

**Example:** Find the center and radius of equation that is not given in standard form.
\[
x^2 + 2x + y^2 + 4y - 5 = 0
\]
\[
(x^2 + 2x + \phantom{0}) + (y^2 + 4y + \phantom{0}) = 5
\]
\[
(x^2 + 2x + 1) + (y^2 + 4y + 4) = 5 + 1 + 4 \quad \text{Complete the square}
\]
\[
(x + 1)^2 + (y + 2)^2 = 10
\]
**Center:** (-1, -2)  **Radius:** \( r = \sqrt{10} \)

**Graphing a Circle:**

If the equation is in standard form, you can easily obtain the information needed to graph the circle. If it is not in standard form, complete the square so you can then write it in standard form.

**Example:**  \((x - 2)^2 + (y - 3)^2 = 16\)
You know the center of this circle is (2, 3), so place that point on your graph first.
You also know that the radius is \( \sqrt{16} = 4 \). Remember the radius is the distance from the center to any point on the circle. So, moving 4 units to the right from the center will give you a point on the circle (6, 3). Likewise, you can move 4 units to the left from the center to obtain (-2, 3), 4 units up from the center to obtain (2, 7), and 4 units down from the center to obtain (2, -1). You now have four points on our circle along with the center to sketch a graph.