Name ________________________________

School ________________________________

SR, JR, SO, or FR ______________________

Missouri Western State College
Calculus Bee
Written Exam – April 18, 2005

Each problem is worth five (5) points. Partial credit may be awarded.

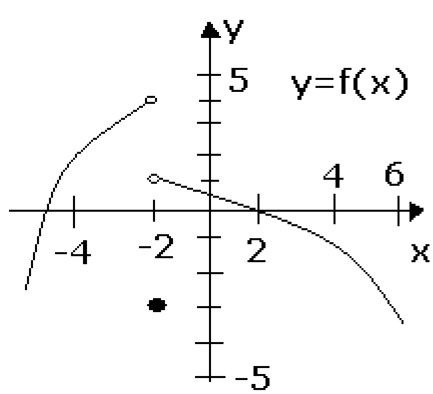
Problem	Points	Grader
1		Jeff
2		Dave
3		Tim
4		Ken
5		Don
6		Jennifer
7		Sharon
8		Sharon
9		Kevin
10		Steve
TOTAL		

1. Find each limit (if it exists).

a. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}=\lim_{x\to 3}\left(\frac{\sqrt{x+1}-2}{x-3}\right)\left(\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}\right)=\lim_{x\to 3}\frac{1}{\sqrt{x+1}+2}=\frac{1}{4}$

b. $\lim_{x\to -\infty}\frac{3x-2}{\sqrt{2x^2+1}}=\lim_{x\to -\infty}\frac{\frac{1}{x}(3x-2)}{\frac{1}{x}\sqrt{2x^2+1}}=\lim_{x\to -\infty}\frac{3-2/x}{-\sqrt{2+1/x^2}}=-\frac{3}{\sqrt{2}}$

c. For the graph of $y=f(x)$ given below, find $\lim_{x\to -2^+} f(x)$. $\lim_{x\to -2^+} f(x)=1$



d. For the function *f*, shown above, is $f'(4)$ positive, zero, or negative?

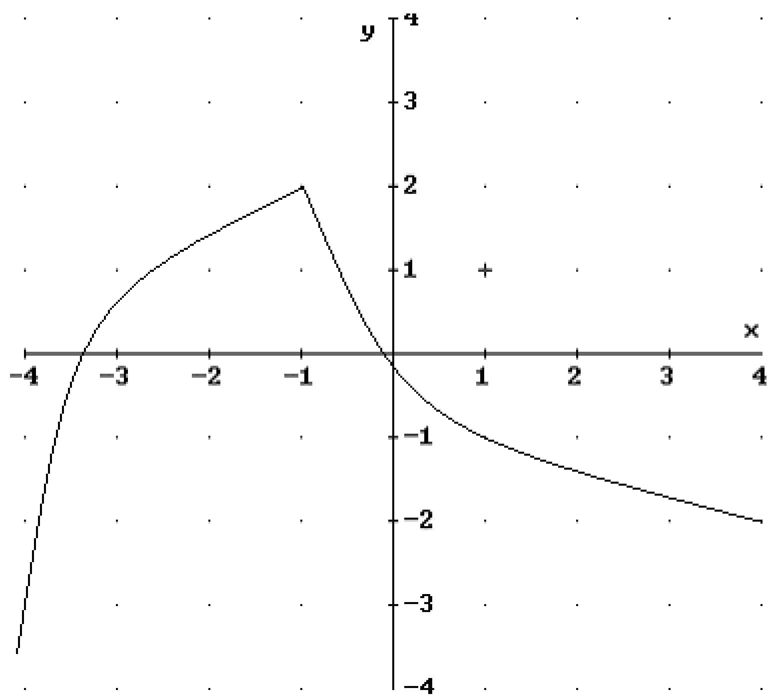
negative

e. For the function *f*, shown above, is $f''(-4)$ positive, zero, or negative?

negative

2. Sketch the graph of a continuous function that satisfies the given conditions.

$$f'(x) \text{ is } \begin{cases} >0, & x<-1 \\ \text{undefined}, & x=-1 \\ <0, & x>-1 \end{cases} \text{ and } f''(x) \text{ is } \begin{cases} <0, & x<-1 \\ \text{undefined}, & x=-1 \\ >0, & x>-1 \end{cases}.$$



3. Determine the derivative of each of the following. DO NOT SIMPLIFY.

a. $y = 3x^4 + 7x^3 + 5x + 10$

$$y' = 12x^3 + 21x^2 + 5$$

b. $y = \left(\dfrac{t^2}{t^3 + 2} \right)^2$

$$y' = 2\left(\frac{t^2}{t^3 + 2} \right) \frac{(t^3 + 2)2t - t^2(3t^2)}{(t^3 + 2)^2}$$

c. $f(\theta) = \theta^2 \tan\theta$

$$f'(\theta) = 2\theta \tan\theta + \theta^2 \sec^2\theta$$

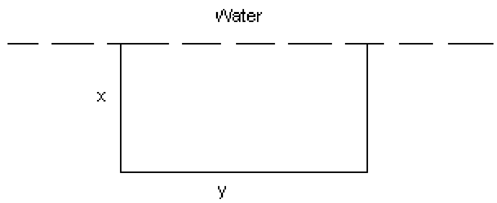
d. $g(x) = \sin^2\left(\dfrac{\pi}{4} \right)$

$$g'(x) = 0$$

e. $F(x) = \displaystyle\int_1^{x^3} \frac{e^t}{t^2}\, dt$

$$F'(x) = \frac{e^{x^3}}{\left(x^3\right)^2} 3x^2$$

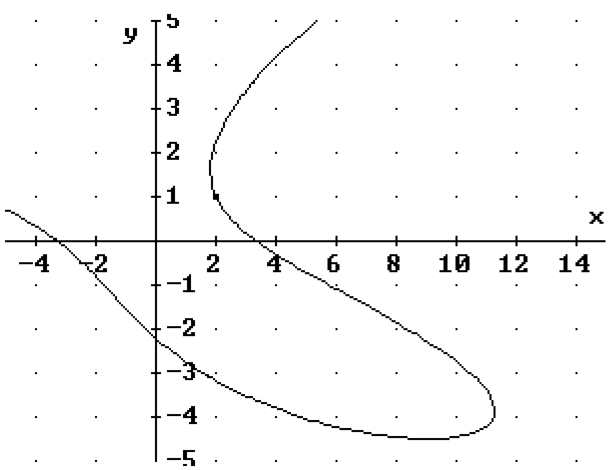
4. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?



$xy = 180,000 \Rightarrow y = \dfrac{180,000}{x}$ Minimize $L = 2x + y = 2x + 180,000x^{-1}$

So, $L' = 2 - 180,000x^{-2} = \dfrac{2(x^2 - 90,000)}{x^2} = 0 \Rightarrow x = \sqrt{90,000} = 300$ and $y = 600$

5. Find the equation, $y = mx + b$, of the tangent line to the curve $x^2 + 4xy - y^3 = 11$ at the point $(2,1)$.

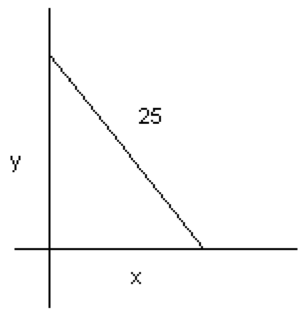


Using implicit differentiation we get $\dfrac{dy}{dx} = \dfrac{2x + 4y}{3y^2 - 4x}$. So the slope of the line is

$m = \left.\dfrac{dy}{dx}\right|_{(2,1)} = \dfrac{4+4}{3-8} = -\dfrac{8}{5}$. Using this slope and the point (2,1) gives the line

$y = -\dfrac{8}{5}x + \dfrac{21}{5}$

6. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when the base of the ladder is 15 feet from the wall?



Here we have $\frac{dx}{dt} = 2$ and $x^2 + y^2 = 25^2 \Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$.

Now, when x = 15 , $225 + y^2 = 625 \Rightarrow y = 20$.

Finally, $\left.\frac{dy}{dt}\right|_{x=15} = -\frac{15}{20}2 = -\frac{3}{2}\,ft/\sec$

7. Find the value of c guaranteed by the Mean Value Theorem for $f(x) = \ln x$ on $[1,4]$.

$\frac{1}{c} = f'(c) = \frac{f(4) - f(1)}{3} = \frac{\ln 4 - \ln 0}{3} = \frac{\ln 4}{c} \Rightarrow c = \frac{3}{\ln 4}$

8. Evaluate each of the following integrals.

a. $\int t^2\left(t-\frac{2}{t}\right)dt$

$$\int (t^3-2t)dt=\frac{1}{4}t^4-t^2+C$$

b. $\int_1^4 \frac{t+1}{t}\,dt$

$$\int_1^4 1+\frac{1}{t}dt=t+\ln t\Big|_1^4=3+\ln 4$$

c. $\int_1^3 \frac{\pi}{4}\,dx$

$$\frac{\pi}{4}(3-1)=\frac{\pi}{2}$$

d. $\int e^{\ln x^2}\cdot x\,dx$

$$\int x^3dx=\frac{1}{4}x^4+C$$

e. $\int \frac{\cos x}{\sqrt{1+\sin x}}dx=\int \frac{du}{\sqrt{u}}=2\sqrt{u}+C=2\sqrt{1+\sin x}+C$

$u=1+\sin x$

$du=\cos x dx$

9. Determine the function *f* such that $f''(x)=2$, $f'(2)=5$, and $f(2)=10$.

$f'(x)=2x+C$ so $5=f'(x)=4+C \Rightarrow C=1$.
Thus $f'(x)=2x+1 \Rightarrow f(x)=x^2+x+C_1$.
Then $10=f(2)=4+2+C_1=6+C_1 \Rightarrow C_1=4$
Finally $f(x)=x^2+x+4$

10. Determine the area of the region bounded by the curves $y=\sqrt{x}$ and $y=\frac{1}{4}x+\frac{3}{4}$.

To find the endpoints set the equations equal to each other.
$\sqrt{x}=\frac{1}{4}x+\frac{3}{4} \Rightarrow 4x=x+3 \Rightarrow 16x=x^2+6x+9 \Rightarrow 0=x^2-10x+9 \Rightarrow x=9,1$
With the above endpoints we have

$$A=\int_1^9 \sqrt{x}-\left(\frac{1}{4}x-\frac{3}{4}\right)dx=\frac{4}{3}$$