

# Graphing Sine and Cosine

First, let's look at the basic graphs of sine and cosine:

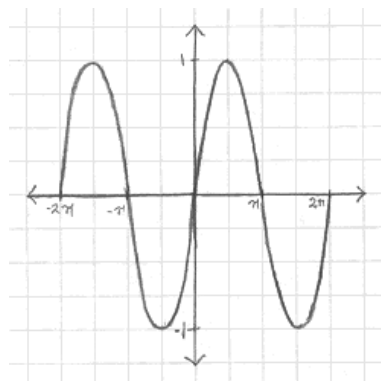


Fig. 1:  $y = \sin x$ , period =  $2\pi$

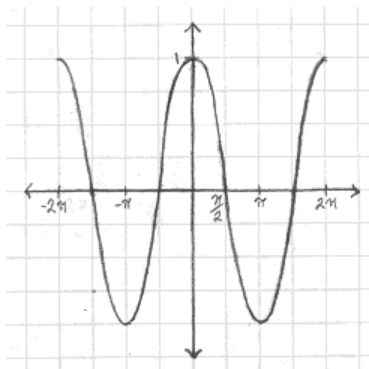


Fig. 2:  $y = \cos x$ , period =  $2\pi$

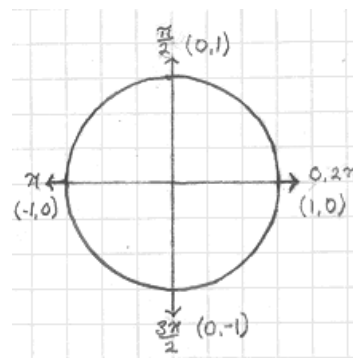


Fig. 3: Unit Circle

Note:

- The sine graph crosses through the origin.
- The sine and cosine graphs each oscillate between  $y = 1$  and  $y = -1$ .
- The ordered pairs for these graphs were derived from the unit circle (Fig. 3).

To graph sine and cosine, use either of the general forms:

$$y = A \sin(B[x - C]) + D \text{ or } y = A \cos(B[x - C]) + D$$

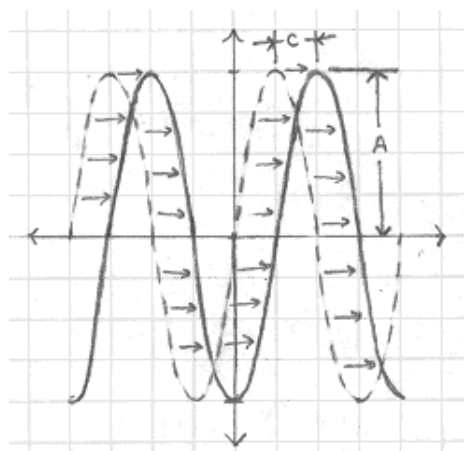


Fig. 4: Sine graph shifted  $C$  units to the right with amplitude  $A$ .

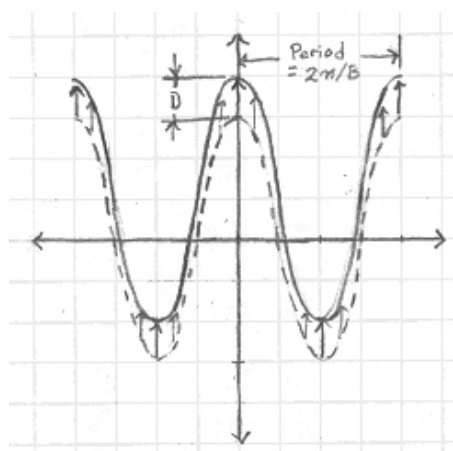


Fig. 5: Cosine graph shifted  $D$  units up with a period of  $2\pi/B$ .

Ex. 1: Determine the amplitude, period, and phase shift. Then graph two cycles of  $y = 3\sin(2x + \pi/2)$ .

1. Rewrite in general form by factoring a 2 out of  $2x + \pi/2$ :  

$$y = 3\sin(2[x - (-\pi/4)])$$

From this equation we get  
 $A = 3$ ,  $B = 2$ , and  $C = -\pi/4$

So, the amplitude is 3, the period is  $2\pi/2$ , which equals  $\pi$ , and the phase shift is  $\pi/4$  units to the left.

Note: If C is positive, shift the graph to the right; if negative, shift to the left.

2. Lightly sketch the graph of  $y = 3\sin(2x)$ .

Note: We are just excluding the phase shifts at this step.

3. Shift the graph  $\pi/4$  units to the left.

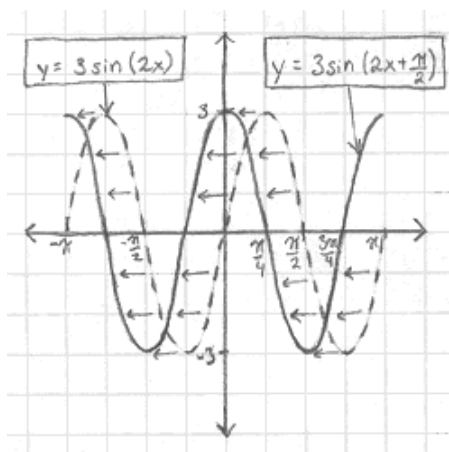


Fig. 6:  $y = 3\sin(2x)$   
 $y = 3\sin(2x + \pi/2)$

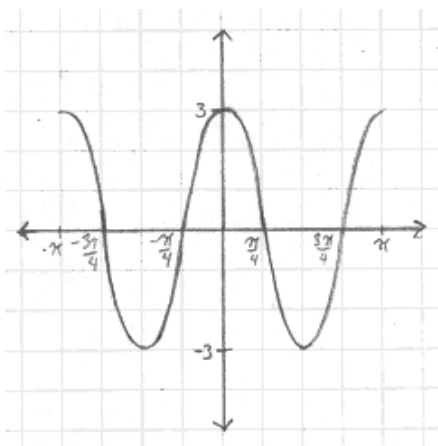


Fig. 7:  $y = 3\sin(2x + \pi/2)$

Ex. 2: Determine the amplitude, period, and phase shift. Then graph two cycles of  $y = -2\cos(\pi x - \pi/2) + 1$ .

1. Rewrite in general form by factoring a  $\pi$  out of  $\pi x - \pi/2$ :  

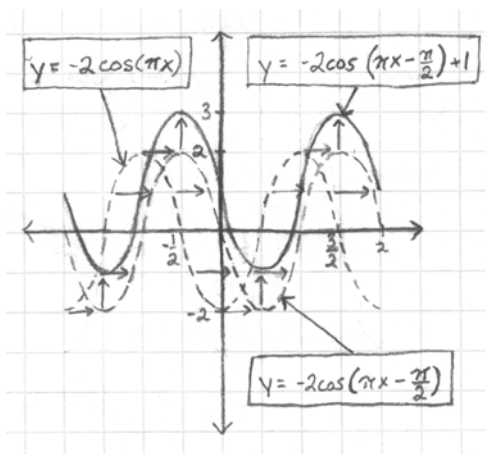
$$y = -2\cos(\pi[x - 1/2]) + 1$$

From this equation we get  
 $A = -2$ ,  $B = \pi$ ,  $C = 1/2$ , and  $D = 1$

So, the amplitude is 2, the period is  $2\pi/\pi$ , which equals 2, and the phase shift is  $1/2$  units to the right and 1 unit up. Since the A coefficient is negative, the graph will be reflected about the x-axis.

2. Lightly sketch the graph of  $y = -2\cos(\pi x)$ .

3. Shift the graph  $1/2$  unit to the right and 1 unit up.



$$y = -2\cos(\pi x)$$

Fig. 8:  $y = -2\cos(\pi x - \pi/2)$

$$y = -2\cos(\pi x - \pi/2) + 1$$

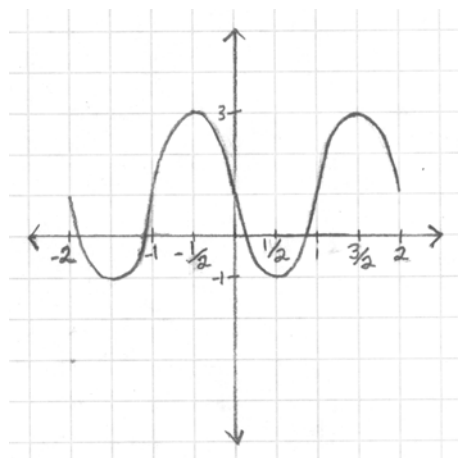
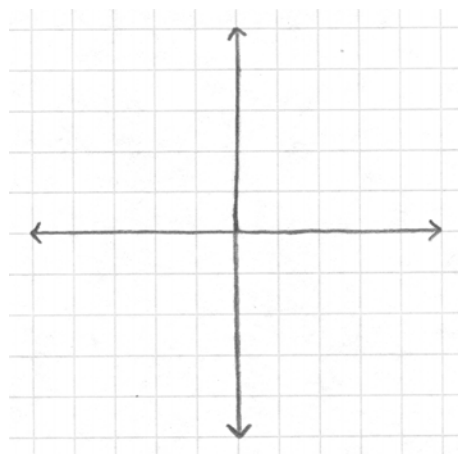
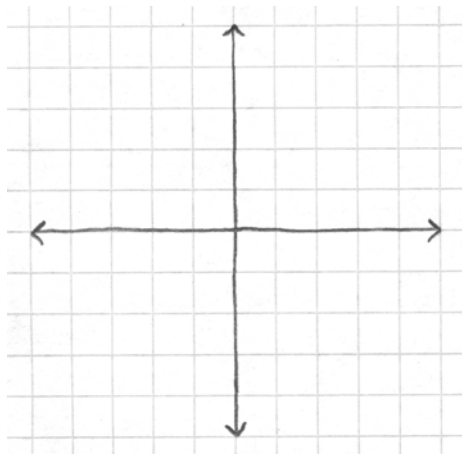


Fig. 9:  $y = -2\cos(\pi x - \pi/2) + 1$

Now try it yourself...

Determine the amplitude, period, and phase shift. Then graph two cycles of

a)  $y = \sin\left(\frac{1}{2}\pi x - \frac{\pi}{2}\right)$       and      b)  $y = 3\cos(4x) - 1$ .



(answers on back)

a) The amplitude is 1, the period is 4, and the phase shift is 1 unit to the right.

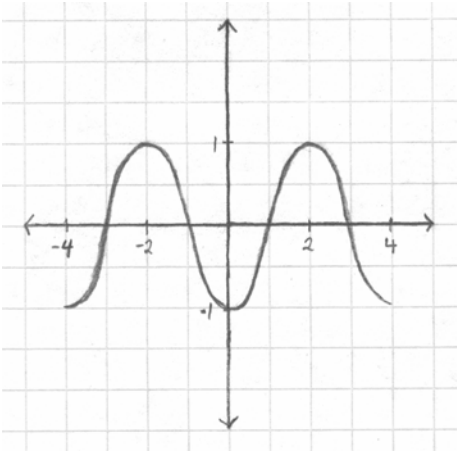


Fig. 10:  $y = \sin\left(\frac{1}{2}\pi x - \frac{\pi}{2}\right)$

b) The amplitude is 3, the period is  $\pi/2$ , and the phase shift is 1 unit down.

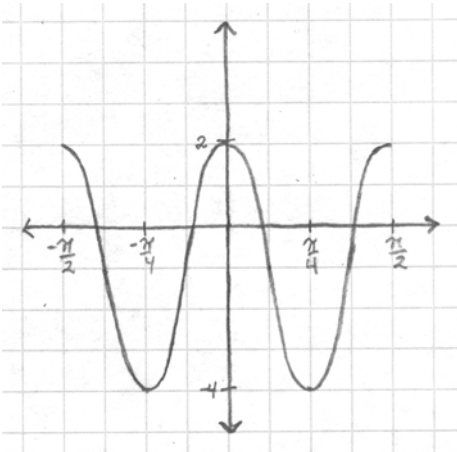


Fig. 11:  $y = 3\cos(4x) - 1$