Graphing Sine and Cosine

First, let's looks at the basic graphs of sine and cosine:

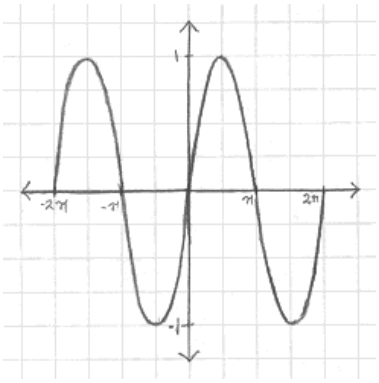


Fig. 1: $y = \sin x$, period = 2π

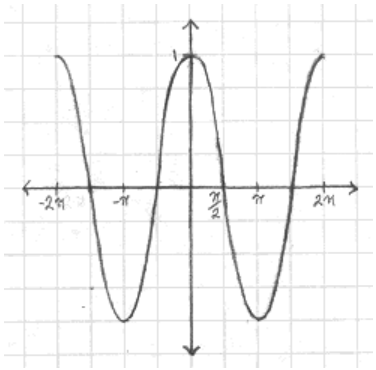


Fig. 2: $y = \cos x$, period = 2π

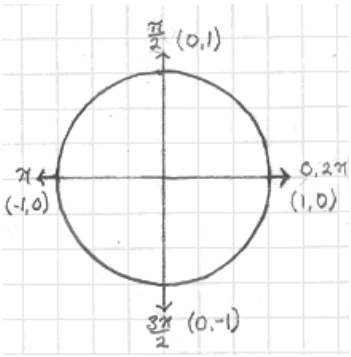


Fig. 3: Unit Circle

Note:

- The sine graph crosses through the origin.
- The sine and cosine graphs each oscillate between y = 1 and y = -1.
- The ordered pairs for these graphs were derived from the unit circle (Fig. 3).

To graph sine and cosine, use either of the general forms:

$$y = A\sin(B[x - C]) + D \text{ or } y = A\cos(B[x - C]) + D$$

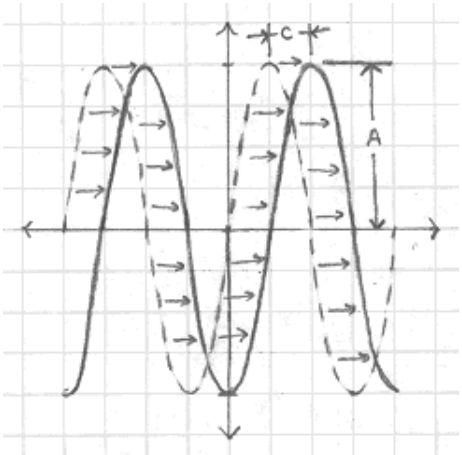


Fig. 4: Sine graph shifted C units to the right with amplitude A.

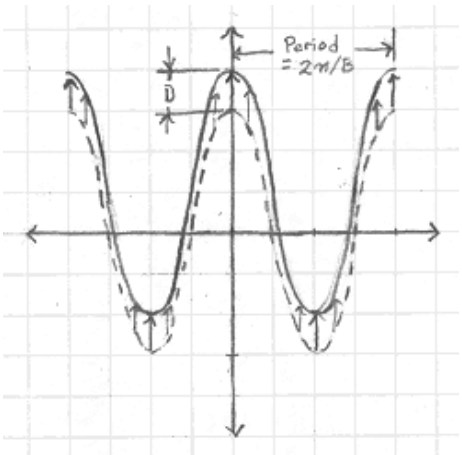


Fig. 5: Cosine graph shifted D units up with a period of 2π /B.

Ex. 1: Determine the amplitude, period, and phase shift. Then graph two cycles of $y = 3\sin(2x + \pi/2)$.

1. Rewrite in general form by factoring a 2 out of 2x + π/2:
   $y = 3\sin(2[x - (-\pi/4)])$

   From this equation we get
   A = 3, B = 2, and C = - π/4

   So, the amplitude is 3, the period is 2π/2, which equals π, and the phase shift is π/4 units to the left.

   Note: If C is positive, shift the graph to the right; if negative, shift to the left.

2. Lightly sketch the graph of $y = 3\sin(2x)$.

   Note: We are just excluding the phase shifts at this step.

3. Shift the graph π/4 units to the left.

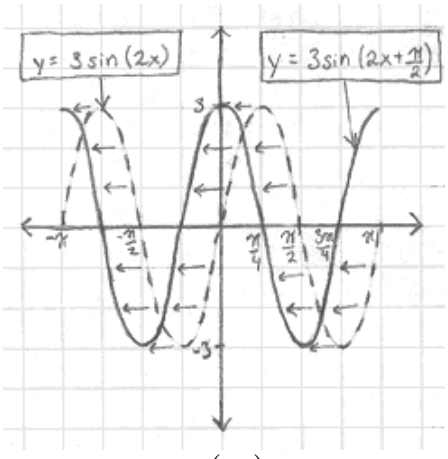


Fig. 6: $y = 3\sin(2x)$
$y = 3\sin(2x + \pi/2)$

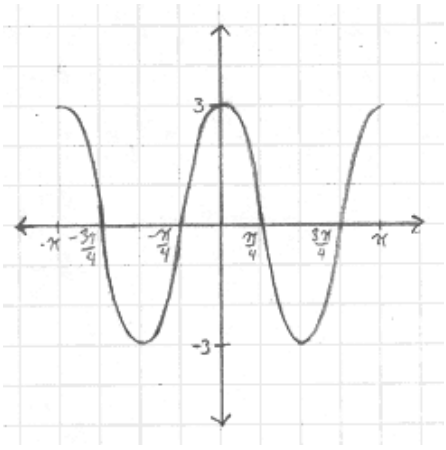


Fig. 7: $y = 3\sin(2x + \pi/2)$

Ex. 2: Determine the amplitude, period, and phase shift. Then graph two cycles of $y = -2\cos(\pi x - \pi/2) + 1$.

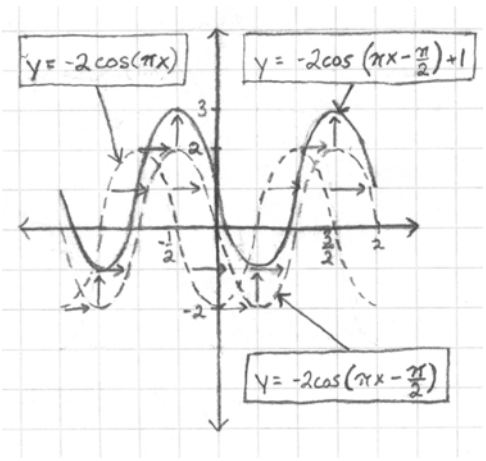
1. Rewrite in general form by factoring a π out of πx- π/2:
   $y = -2\cos(\pi[x - 1/2]) + 1$

   From this equation we get
   A = - 2, B = π, C = 1/2, and D = 1

   So, the amplitude is 2, the period is 2π/π, which equals 2, and the phase shift is 1/2 units to the right and 1 unit up. Since the A coefficient is negative, the graph will be reflected about the x-axis.

2. Lightly sketch the graph of $y = -2\cos(\pi x)$.

3. Shift the graph 1/2 unit to the right and 1 unit up.



$y = -2\cos(\pi x)$

Fig. 8: $y = -2\cos(\pi x - \pi/2)$

$y = -2\cos(\pi x - \pi/2) + 1$

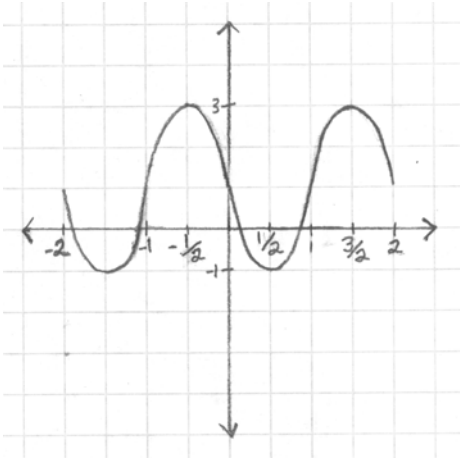
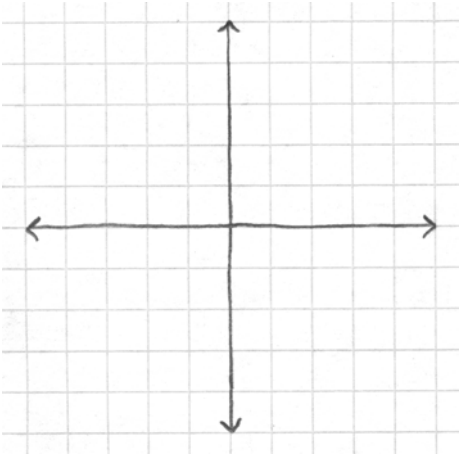


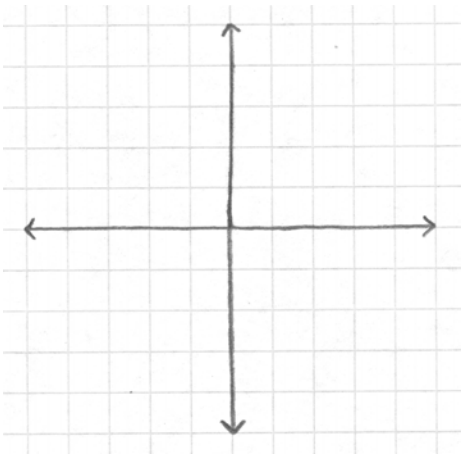
Fig. 9: $y = -2\cos(\pi x - \pi/2) + 1$

Now try it yourself…

Determine the amplitude, period, and phase shift. Then graph two cycles of

a) $y = \sin\left(\frac{1}{2}\pi x - \frac{\pi}{2}\right)$ and b) $y = 3\cos(4x) - 1$.





(answers on back)

a) The amplitude is 1, the period is 4, and the phase shift is 1 unit to the right.

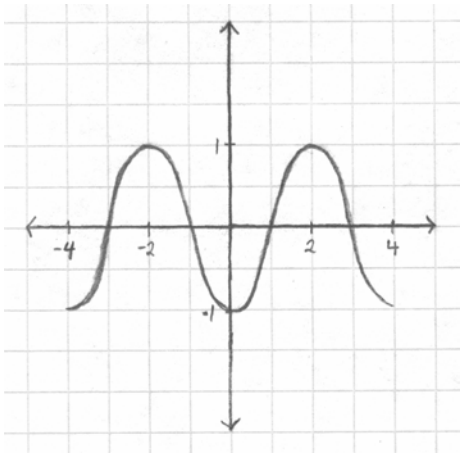


Fig. 10: $y = \sin\left(\frac{1}{2}\pi x - \frac{\pi}{2}\right)$

b) The amplitude is 3, the period is π/2, and the phase shift is 1 unit down.

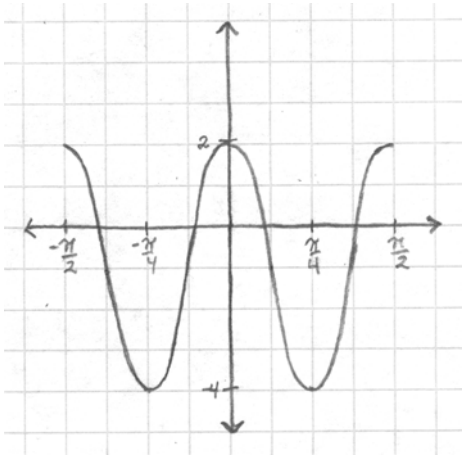


Fig. 11: $y = 3\cos(4x) - 1$