Function Arithmetic

Just as we are able to add (+), subtract (-), multiply (×), and divide (÷) two or more numbers, we are able to + , - , × , and ÷ two or more functions. Let's look at some basic notation before we go any further.

$$f(x) + g(x)$$

This means that we will add the contents of $f(x)$ to the contents of $g(x)$. Another way it can be written is $(f + g)(x)$, which is more common to see. The remaining operations follow similar notation:

$f(x) - g(x) = (f - g)(x)$

$f(x) \cdot g(x) = (f \cdot g)(x)$

$\frac{f(x)}{g(x)} = \left(\frac{f}{g}\right)(x)$

Let's use the following two functions to demonstrate how the operations work:

$$f(x) = 3x + 2 \qquad\qquad g(x) = -2x + 1$$

There are two ways to calculate values when we + , - , × , and ÷ functions. **(1)** We can substitute a given number for x in each function first, and then perform the desired operation with the results of each function, or **(2)** we can perform the operation on the functions first, and then substitute the given number for x. Let's work some examples of each operation. We'll first use way **(1)** to find our answer in the left column, and then we will use way **(2)** to find our answer in the right column.

$$(f + g)(-1)$$

$f(-1) = 3(-1) + 2 = -3 + 2 = -1$	$f(x) + g(x) = (3x + 2) + (-2x + 1)$
$g(-1) = -2(-1) + 1 = 2 + 1 = 3$	$3x + 2 - 2x + 1 = x + 3$
$f(-1) + g(-1) = -1 + 3 = 2$	$(f + g)(-1) = (-1) + 3 = 2$

$$(g - f)(2)$$

$f(2) = 3(2) + 2 = 6 + 2 = 8$	$g(x) - f(x) = (-2x + 1) - (3x + 2)$
$g(2) = -2(2) + 1 = -4 + 1 = -3$	$-2x + 1 - 3x - 2 = -5x - 1$
$g(2) - f(2) = -3 - 8 = -11$	$(g - f)(2) = -5(2) - 1 = -11$

$(f \cdot g)(3)$

$f(3) = 3(3) + 2 = 9 + 2 = 11$

$g(3) = -2(3) + 1 = -6 + 1 = -5$

$f(3) \cdot g(3) = 11 \cdot -5 = -55$

$f(x) \cdot g(x) = (3x + 2)(-2x + 1)$

$-6x^2 + 3x - 4x + 2 = -6x^2 - x + 2$

$(f \cdot g)(3) = -6(3)^2 - 3 + 2 = -55$

$\left(\frac{f}{g}\right)(-3)$

$f(-3) = 3(-3) + 2 = -9 + 2 = -7$

$g(-3) = -2(-3) + 1 = 6 + 1 = 7$

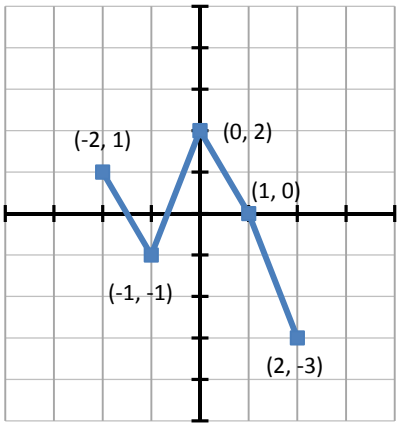
$\frac{f(-3)}{g(-3)} = \frac{-7}{7} = -1$

$\frac{f(x)}{g(x)} = \frac{3x + 2}{-2x + 1}$

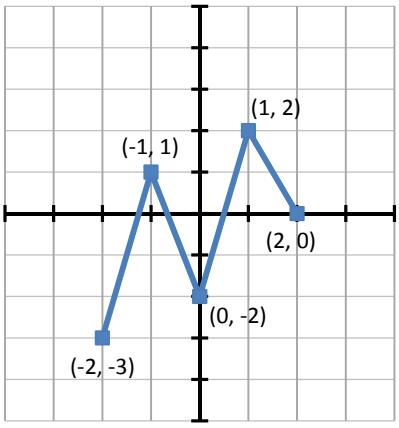
$\left(\frac{f}{g}\right)(-3) = \frac{3(-3) + 2}{-2(-3) + 1} = \frac{-7}{7} = -1$

Not only are we able to combine function equations together with arithmetic operations, we are able to combine function graphs together with the same operations. Consider the following two functions:

$f(x) =$



$g(x) =$



Let's find $(f + g)(x)$ and $(f - g)(x)$. The first thing we need to do is to create a table with the ordered pairs of the points on each graph.

$f(x) =$

x	-2	-1	0	1	2
y	1	-1	2	0	-3

$g(x) =$

x	-2	-1	0	1	2
y	-3	1	-2	2	0

The next thing we do is find **matching x values**. For each matching x value we have, we perform the desired operation on the **corresponding y values**. For $(f+g)(x)$:

$(f+g)(x) =$

x	-2	-1	0	1	2
y	(1) + (-3)	(-1) + (1)	(2) + (-2)	(0) + (2)	(-3) + (0)

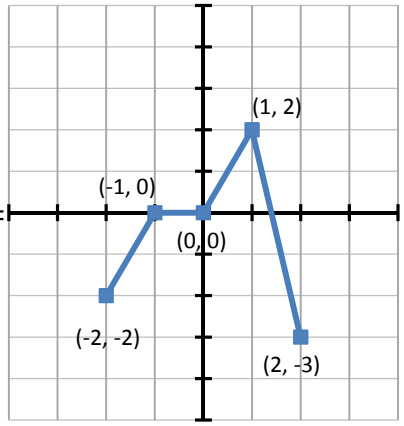
Giving us:

$(f+g)(x) =$

x	-2	-1	0	1	2
y	-2	0	0	2	-3

and the final graph:

$(f+g)(x) =$



For $(f-g)(x)$:

$(f-g)(x) =$

x	-2	-1	0	1	2
y	(1) - (-3)	(-1) - (1)	(2) - (-2)	(0) - (2)	(-3) - (0)

Giving us:

$(f-g)(x) =$

x	-2	-1	0	1	2
y	4	-2	4	-2	-3

and the final graph:

$(f-g)(x) =$