

## Fractions

Numbers such as  $\frac{1}{2}$ ,  $\frac{1}{4}$  and  $\frac{1}{3}$  are fractions.

When dealing with fractions, the top number is called the numerator and the bottom number is called the denominator.

1 numerator  
2 denominator

When solving fractions, the numerator is being divided by the denominator.

$$1 \div 2 \text{ or } 2 \overline{)1}$$

There are a few different types of fractions including proper fractions, improper fractions and mixed numbers.

When the denominator is *greater* than the numerator, it is considered a proper fraction.

Proper fractions are numbers like  $\frac{3}{7}$ ,  $\frac{4}{5}$  and  $\frac{11}{16}$ .

Fractions can also be negative numbers like  $-\frac{5}{8}$ ,  $-\frac{2}{9}$  and  $-\frac{7}{-16}$ .

When the denominator is *smaller* than the numerator, it is considered an improper fraction.

Improper Fractions are numbers like  $\frac{4}{3}$ ,  $\frac{7}{5}$  and  $-\frac{13}{8}$ .

When there is a whole number and a fraction, it is considered a mixed number.

Mixed numbers are numbers like  $1\frac{1}{2}$ ,  $3\frac{2}{5}$  and  $-5\frac{3}{8}$ .

To convert a mixed number to an improper fraction, multiply the whole number by the denominator and add the result to the numerator. The denominator will stay the same.

$$\text{Ex: } 2\frac{3}{8} = \frac{(2)(8) + 3}{8} = \frac{16 + 3}{8} = \frac{19}{8}$$

To convert an improper fraction to a mixed number, divide the numerator by the denominator. The remainder will be the numerator of the fraction and the denominator will stay the same.

$$\text{Ex: } \frac{22}{7} = 7 \overline{)22} = 3\frac{1}{7}$$

## Adding and Subtracting Fractions

### Adding Fractions:

If fractions have the same denominator, the numerators can be added together. The denominators remain the same.

An Example:

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$$

If fractions do not have the same denominator, a common multiple of the denominators must be found before the fractions can be added together.

A common multiple can be defined as any number that is divisible by two different values. In this case, a common multiple is needed for the two different denominator values.

An Example:

$$\frac{2}{5} + \frac{1}{3} =$$

The multiples of 5 are  $1 \times 5 = 5$ ,  $2 \times 5 = 10$ ,  $3 \times 5 = 15$ ,  $4 \times 5 = 20$ ,  $5 \times 5 = 25 \dots$

The multiples of 3 are  $1 \times 3 = 3$ ,  $2 \times 3 = 6$ ,  $3 \times 3 = 9$ ,  $3 \times 4 = 12$ ,  $3 \times 5 = 15$ ,  $3 \times 6 = 18 \dots$

A common multiple of 3 and 5 is 15.

The numerator and the denominator of each fraction must be multiplied by the same number in order for the fractions to be added.

$$\frac{(3)(2)}{(3)(5)} + \frac{(5)(1)}{(5)(3)} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}$$

1) Try this example:

$$\frac{4}{9} + \frac{3}{4} = \text{(See page 4 for solution)}$$

### Subtracting Fractions:

Subtracting fractions is very similar to adding fractions. When subtracting fractions with equivalent denominators, subtract the numerators and use the same denominator.

An Example:

$$\frac{6}{10} - \frac{3}{10} = \frac{3}{10}$$

If fractions do not have the same denominator, a common multiple of the denominators must be found before the fractions can be subtracted.

An Example:

$$\frac{3}{4} - \frac{4}{7} =$$

The multiples of 4 are:  $1 \times 4 = 4$ ,  $2 \times 4 = 8$ ,  $3 \times 4 = 12$ ,  $4 \times 4 = 16$ ,  $5 \times 4 = 20$ ,  $6 \times 4 = 24$ ,  
 $7 \times 4 = 28$ ,  $8 \times 4 = 32 \dots$

The multiples of 7 are:  $1 \times 7 = 7$ ,  $2 \times 7 = 14$ ,  $3 \times 7 = 21$ ,  $4 \times 7 = 28$ ,  $5 \times 7 = 35 \dots$

A common multiple of 4 and 7 is 28.

The numerator and the denominator of each fraction must be multiplied by the same number in order for the fractions to be subtracted.

$$\frac{(7)(3)}{(7)(4)} - \frac{(4)(4)}{(4)(7)} = \frac{21}{28} - \frac{16}{28} = \frac{5}{28}$$

2) Try this example:

$$\frac{5}{8} - \frac{2}{5} = \text{(see page 4 for solution)}$$

### Adding and Subtracting Mixed Numbers

To add and subtract mixed numbers, the first step is to convert the mixed numbers to improper fractions. Then add or subtract them as fractions.

An Example:

$$3\frac{5}{8} + 5\frac{1}{2} =$$

$$\text{Convert: } 3\frac{5}{8} = \frac{(3)(8) + 5}{8} = \frac{24 + 5}{8} = \frac{29}{8}$$

$$\text{Convert: } 5\frac{1}{2} = \frac{(5)(2) + 1}{2} = \frac{10 + 1}{2} = \frac{11}{2}$$

$$\text{Rewrite: } \frac{29}{8} + \frac{11}{2}$$

The multiples of 8 are:  $1 \times 8 = 8$ ,  $2 \times 8 = 16$ ,  $3 \times 8 = 24 \dots$

The multiples of 2 are:  $1 \times 2 = 2$ ,  $2 \times 2 = 4$ ,  $3 \times 2 = 6$ ,  $4 \times 2 = 8$ ,  $5 \times 2 = 10 \dots$

The common multiple of 8 and 2 is 8. The fraction  $\frac{29}{8}$  has an 8 in the denominator; therefore a 1 is used in the multiplication for a common denominator.

$$\frac{(1)(29)}{(1)(8)} + \frac{(4)(11)}{(4)(2)} = \frac{29}{8} + \frac{44}{8} = \frac{73}{8}$$

An Example:

$$4\frac{2}{5} - 6\frac{4}{7} =$$

$$\text{Convert: } 4\frac{2}{5} = \frac{(4)(5) + 2}{5} = \frac{20 + 2}{5} = \frac{22}{5} \quad \text{Convert: } -6\frac{4}{7} = -\frac{(6)(7) + 4}{7} = -\frac{42 + 4}{7} = -\frac{46}{7}$$

$$\text{Rewrite: } \frac{22}{5} - \frac{46}{7}$$

The multiples of 5 are  $1 \times 5 = 5, 2 \times 5 = 10, 3 \times 5 = 15, 4 \times 5 = 20, 5 \times 5 = 25, 6 \times 5 = 30,$   
 $7 \times 5 = 35, 8 \times 5 = 40\dots$

The multiples of 7 are:  $1 \times 7 = 7, 2 \times 7 = 14, 3 \times 7 = 21, 4 \times 7 = 28, 5 \times 7 = 35, 6 \times 7 = 42\dots$

The common multiple of 5 and 7 is 35.

$$\frac{(7)(22)}{(7)(5)} - \frac{(5)(46)}{(5)(7)} = \frac{154}{35} - \frac{230}{35} = -\frac{76}{35}$$

3) Try this example:

$$4\frac{9}{11} - 7\frac{3}{2} = \text{(See below for solution)}$$

**Solutions:**

1)  $\frac{43}{36}$

2)  $\frac{9}{40}$

3)  $-\frac{81}{22}$