

What You Need to Know About a Straight Line

A. Necessary Concepts, Forms of the Equation, and Formulas:

1. The slope of a line is the tilt or slant.

$$2. \quad \text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

3. Standard form for the equation of a line is:

$Ax + By = C$ (notice this is just the x term first, then the y term, then the equals sign, and finally the constant)

ex: $3x - 5y = 7$

4. The equation of a line may also be written in slope-intercept form:

$$y = mx + b \text{ (notice this equation is solved for } y)$$

ex: $y = -2x + 4$

5. Lines with a positive slope rise from lower left to upper right.
Lines with a negative slope fall from upper left to lower right.

6. Two lines are special:

Vertical Lines have the form $x = a$ and have no slope number.

Horizontal Lines have the form $y = b$ and a slope of zero.

ex: $x = 5$ will be vertical and have no slope.

ex: $y = -3$ will be horizontal and have a slope of zero.

7. The point-slope form of the equation is most often used to find the equation of a given line:

$$y - y_1 = m(x - x_1)$$

If you know one point and the slope, use the point slope form to determine the equation of the line.

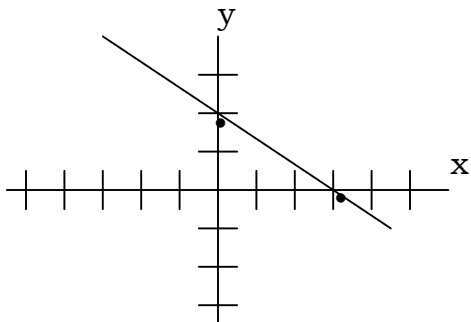
8. Parallel lines have the same slope (they are slanted the same).

Perpendicular lines have slopes that are negative reciprocals of each other. This means the slopes must meet two conditions: opposite in sign and “flipped.”

Following are the things you should be able to do concerning the straight line:

B. Find the slope, given:

1. the graph



Choose two definite points the line passes through. In our example, they are heavy dots.

Note the directions you must travel to move from one point to the other. Let's move from the top left one to the lower right.

We would have to move down 2, then right 3. This represents a change in y of -2 (down) and a change in x of +3 (right).

Therefore, the slope is $-\frac{2}{3}$, or just $-\frac{2}{3}$.

2. two points

ex: given (-2,3) and (4,7), just apply the formula for the slope:

Look at the two points in this way:

x_1, y_1 x_2, y_2
(-2, 3) and (4, 7) and apply the formula for the slope.

$$y = \frac{7-3}{4-(-2)} = \frac{4}{6} = \frac{2}{3}$$

3. the equation in slope-intercept form

If the equation is given to you in slope-intercept form, you're in luck! You can read the slope directly from the equation.

ex: given $y = 3x - 7$, note how this fits slope-intercept form shown below:
 $y = mx + b$

Therefore, the slope is 3.

4. the equation in standard form: $Ax + By = C$

The slope will be $-\frac{A}{B}$.

ex: Given the equation of a line to be $3x - 6y = 7$, the slope will be

$m =$

$$\frac{-A}{B} = \frac{-(3)}{-6} = \frac{1}{2}$$

(The Straight Line, Cont.)

C. Find the equation, given:

1. One point and the slope.

Use the point-slope form of the equation!

ex: Given that a line has slope of -3 and passes through (-4,5), find the equation.

Substituting into the point-slope form gives us:

$$y - 5 = -3[x - (-4)]$$

$$y - 5 = -3[x + 4]$$

$$y - 5 = -3x - 12$$

At this point, we can put the equation in either standard form or slope-intercept form.

In standard form we would have $3x + y = -7$

In slope-intercept form we would have $y = -3x - 7$

Check your homework problem to see which form to use.

2. Two points.

Use the slope formula to find the slope, then use this slope with either of the two points in the point-slope form of the equation as shown above.

3. The slope and the y-intercept.

Simply substitute both into the slope-intercept form of the equation.

Ex: Given the slope of a line is $2/3$ and the y-intercept is -4, find the equation.

Since $m = 2/3$ and $b = -4$, we may substitute into

$$y = mx + b$$

$$y = \frac{2}{3}x - 4$$

4. Given x- and y-intercepts.

If the x-intercept is -3 and the y-intercept is 7, remember the actual coordinates of these points are then (-3,0) and (0,7). What you have really been given are two points so work the problem as in #2 above.

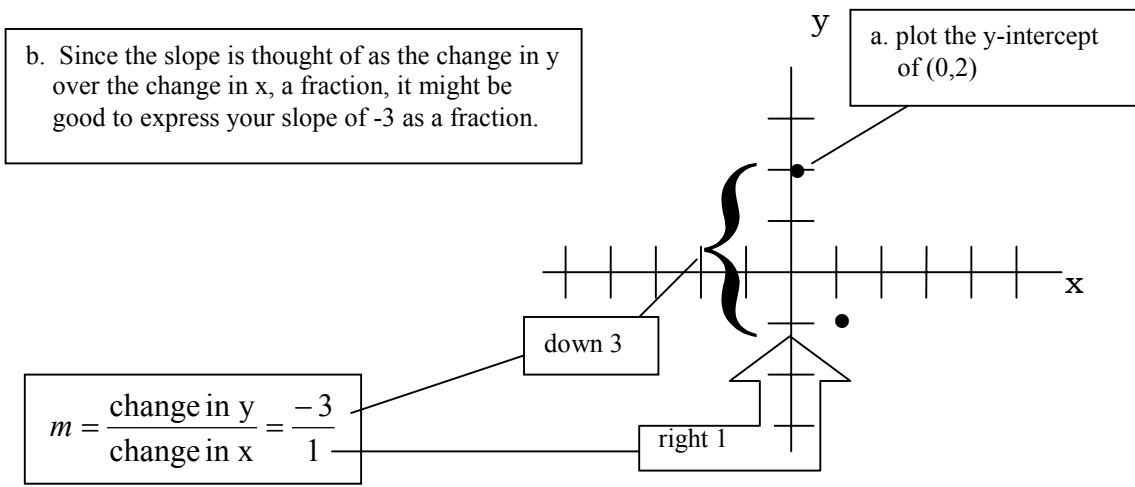
D. Graphing the equation, given:

1. The slope and the y-intercept.

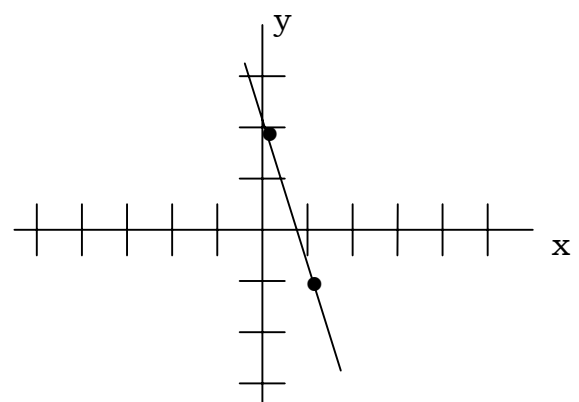
Follow these two steps:

- a. plot the point which is the y-intercept
- b. use the slope as directions to a new point

ex: Graph the line with the slope of -3 and y-intercept of 2.



The line can now be drawn connecting the two points.



2. Given one point and the slope of a line.

Use the same procedure as above. Plot the given point, then use the slope as directions to a new point.