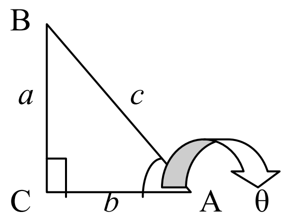
Trigonometry Review

Consider a right angle triangle



Let $\angle BAC = \theta$,
Side AB = c and side BC = a.
We can calculate Side AC = b, by Pythagorean Theorem

$$a^2 + b^2 = c^2$$

<u>We can define 6 trigonometric ratios :</u>

$$Sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{a}{c} \qquad Csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{c}{a}$$

$$Cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{b}{c} \qquad Sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{c}{b}$$

$$Tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{a}{b} \qquad Cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{b}{a}$$

<u>From these definitions we can derive:</u>

$$Sin\theta = \frac{1}{Csc\theta} \qquad Csc\theta = \frac{1}{Sin\theta}$$

$$Cos\theta = \frac{1}{Sec\theta} \qquad Sec\theta = \frac{1}{Cos\theta}$$

$$Tan\theta = \frac{1}{Cot\theta} \qquad Cot\theta = \frac{1}{Tan\theta}$$

Also,

$$Tan\theta = \frac{Sin\theta}{Cos\theta} \qquad Cot\theta = \frac{Cos\theta}{Sin\theta}$$

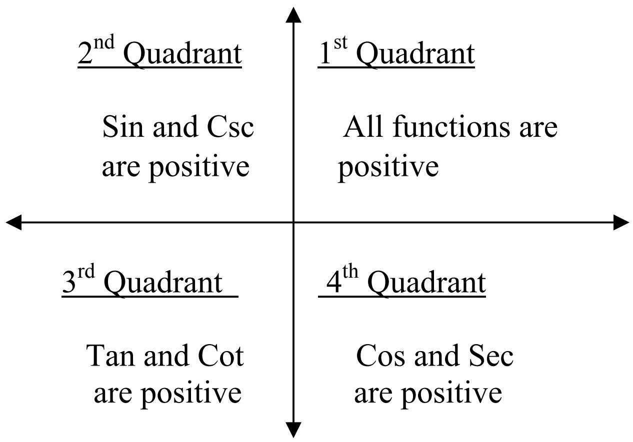
<u>For Example</u>

We have previously defined

$$Sin\theta = \frac{a}{c} \quad , \quad \frac{1}{Sin\theta} = \frac{c}{a} = Csc\theta$$

This can be done for the other trigonometric functions as well.

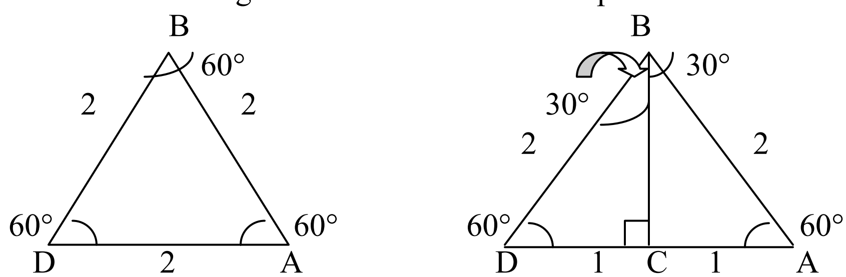
Signs of function values



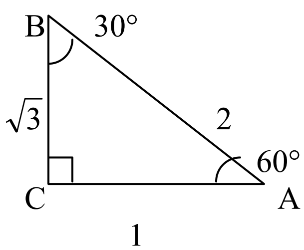
A trick to remember this would be to remember the phrase "All Silver Tea Cups" or "A Smart Trig Class," starting in the first quadrant.

To find function values of special angles

Consider an equilateral triangle, length of each side being 2 units (see figure).
Let us bisect this triangle as shown. We will end up with two 30°-60° triangles.



Now let us consider one of the bisected triangles



By Pythagorean theorem,

$1^2 + (BC)^2 = 2^2$, $(BC)^2 = 4 - 1 = 3$

Therefore, $BC = \sqrt{3}$

We can no find values for 30°.

$$Sin\ 30° = \frac{OppositeSide}{Hypotenuse} = \frac{1}{2} \qquad Cos\ 30° = \frac{AdjacentSide}{Hypotenuse} = \frac{\sqrt{3}}{2}$$

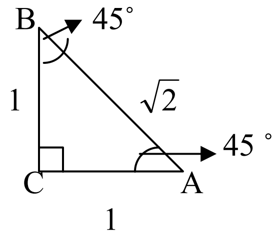
$$Tan\ 30° = \frac{OppositeSide}{AdjacentSide} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Similarly we can find values for 60°.

$$Sin\ 60° = \frac{OppositeSide}{Hypotenuse} = \frac{\sqrt{3}}{2} \qquad Cos\ 60° = \frac{AdjacentSide}{Hypotenuse} = \frac{1}{2}$$

$$Tan\ 60° = \frac{OppositeSide}{AdjacentSide} = \sqrt{3}$$

To find values for 45°, we consider a 45°- 45° triangle



By Pythagorean Theorem

$(AB)^2 = 1^2 + 1^2 = 2$

$AB = \sqrt{2}$

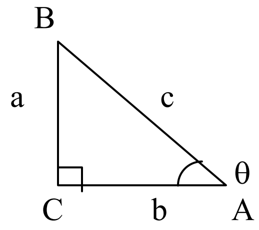
$$Sin\ 45° = \frac{OppositeSide}{Hypotenuse} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad Cos\ 45° = \frac{AdjacentSide}{Hypotenuse} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$Tan\ 45° = \frac{OppositeSide}{AdjacentSide} = 1$$

Table for the most commonly used angles

Angle θ	Sine	Cosine	Tangent
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	Undefined
180°	0	-1	0

Let us now derive some trigonometric identities:



By the Pythagorean Theorem,

$a^2 + b^2 = c^2$ (1.1)

From earlier definitions we have

$Sin\theta = \frac{a}{c}$, $Cos\theta = \frac{b}{c}$, $Tan\theta = \frac{a}{b}$, $Csc\theta = \frac{c}{a}$, $Sec\theta = \frac{c}{b}$, $Cot\theta = \frac{b}{a}$

The three trigonometric identities:

Let us divide equation 1.1 by c^2,
We get

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$$

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

$(Sin\theta)^2 + (Cos\theta)^2 = 1$ or $Sin^2\theta + Cos^2\theta = 1$

If we divide equation 1.1 by a^2 we get,

$$1 + \frac{b^2}{a^2} = \frac{c^2}{a^2}$$

$1 + (Cot\theta)^2 = (Csc\theta)^2$ or $1 + Cot^2\theta = Csc^2\theta$

Finally we divide equation 1.1 by b^2 we get,

$$\frac{a^2}{b^2} + 1 = \frac{c^2}{b^2}$$

$(Tan\theta)^2 + 1 = (Sec\theta)^2$ or $Tan^2\theta + 1 = Sec^2\theta$