Solving Exponential & Logarithmic Equations

BASE $^{\text{EXPONENT}}$ = SOLUTION   is the same as   log $_{\text{BASE}}$ SOLUTION = EXPONENT

I. To Solve Exponential Equations (variable in exponent position):

A. When the bases are the same:

Solve: $3^{X+4} = 3^{2X-1}$

When the bases are the same,
Set the exponents equal to each other
And solve for the variable.
Check the solution by substitution

$x + 4 = 2x - 1$
$-x = -5$
$x = 5$

Check: $3^{(5)+4} = 3^{2(5)-1}$
$3^9 = 3^{10-1}$
$3^9 = 3^9$

B. When the bases are not the same and **NOT** e:

Solve: $3^{X+4} = 5^{X-6}$

Take the log of both sides
Move the exponents in front of the log
Evaluate the log of the constant
Use the distribution property and solve for x

$\log 3^{X+4} = \log 5^{X-6}$
$(x + 4)\log 3 = (x - 6)\log 5$
$x \log 3 + 4 \log 3 = x \log 5 - 6 \log 5$
$x \log 3 - x \log 5 = -4 \log 3 - 6 \log 5$
$x (\log 3 - \log 5) = -1 (4 \log 3 + 6 \log 5)$

$$x = \frac{-1(4\log 3 + 6\log 5)}{\log 3 - \log 5}$$

C. When the base is e:

Solve: $e^{2x - 5} = 29$

Take the ln (natural log) of both sides. Since the base is e and ln is the inverse of e remove (ln e) from one side, evaluate the other solve for x.

$\ln e^{2x - 5} = \ln 29$

$2x - 5 = \ln 29$

$2x = \ln 29 + 5$

$x = \frac{(\ln 29) + 5}{2}$

II. To Solve Logarithmic Equations (log or ln):

A. When every term has the word log (or ln):

Solve: $\log (x - 3) + \log x = \log 18$

Using properties of logarithms, combine logs into one term, both sides of equation have the same base therefore, we can cancel the logs and solve for the indicated variable.

$\log [x(x - 3)] = \log 18$

$x(x - 3) = 18$

$x^2 - 3x - 18 = 0$

$(x - 6)(x + 3) = 0$

$x = 6, x = -3$

Since it is impossible to take the log of a negative number, we disregard the -3 and check the 6.

Check: $\log (6) + \log [(6) - 3] = \log 18$

$\log [6 * (6 - 3)] = \log 18$

$\log (6 * 3) = \log 18$

$\log 18 = \log 18$

B. When not every term has the word log (or ln):

Solve: $\log(x + 2) - \log x = 2$

Using properties of logarithms, combine logs into one term, raise both sides of the equation as a power of the same base. Because the base is the same as the base of the log, we can cancel the log and solve for the indicated variable.

$$\log\left[\frac{(x + 2)}{x}\right] = 2$$

$$10^{\log[(x + 2)/x]} = 10^{2}$$

$$\frac{(x + 2)}{x} = 100$$

$$x + 2 = 100x$$

$$2 = 99x$$

$$\frac{2}{99} = x$$

Check: $\log(2/99 + 2) - \log(2/99) = 2$

$\log\{[(2/99) + 2] / (2/99)\} = 2$

$\log[(2 + 198)/2] = 2$

$\log(200/2) = 2$

$\log 100 = 2$

$2 = 2$