Shifting, Flipping, Stretching, and Compressing Basic Shapes

CENTER FOR ACADEMIC SUPPORT • LRC 213 • 271 – 4524

A. Introduction:

As we examine how two “things” are related to each other, we hope to end up with a statement of their relationship. This statement of how they are related is quite often san equation. And, a very common way of stating how they are related is to use function notation. So, instead of writing “y = 2x + 1, “ we may write f(x) = 2x + 1.

So, we might take a various measurements to determine the relationship between the two things. For instance, we might want to determine how high a projectile might go as related to the elapsed time. If we can determine the relationship, we might state the relationship as a function, like $s(t) = 50 + 16t^2$, where s(t) is the height at time t.

Often, the relationship we discover will be one of several common relationships (or functions). If we look at the graph of one of these common relationships in its simplest forms, then we are looking at the **BASIC SHAPE** of the relationship.

B. Four Basic Shapes:

We will examine four basic relationships and the basic shapes associated with each.

1. A Quadratic Function:

A quadratic function is recognized by the fact that there will be an x^2 term and that this will be the highest power of x present in the function.

The most basic quadratic function would then have to be:

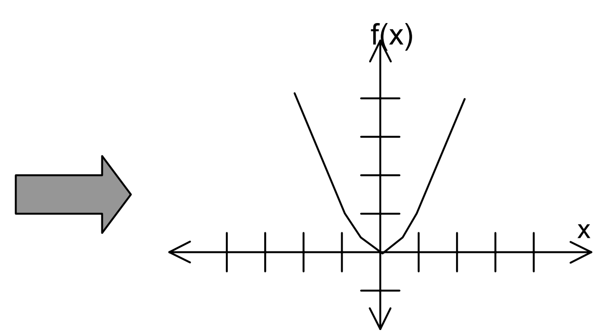
$$f(x) = x^2$$

(Note: There is no way to make this any simpler and still have quadratic function.)

If we make a table of values in order to graph $f(x) = x^2$, it might look like this:

x	f(x)
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

And, be graphed like this

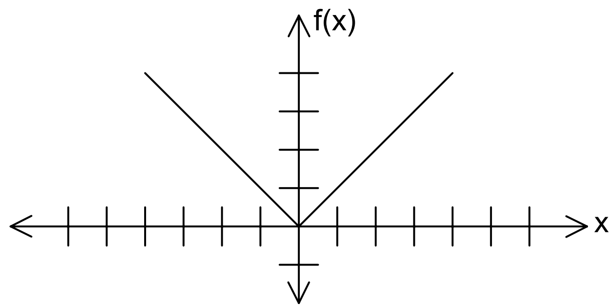


2. An Absolute Value Function:

The absolute value function may be easily recognized since it will have absolute value bars:

$$f(x) = |x|$$

If you were to make a table of values and graph, this basic shape would look this:

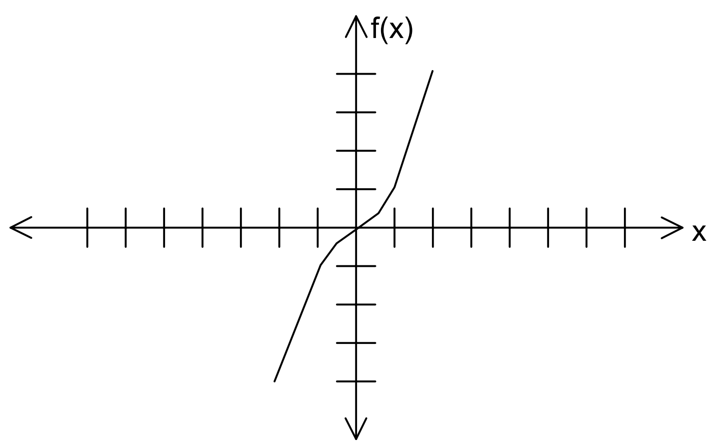


3. The Cubic Function:

The cubic function will look similar to a quadratic function except the degree will be three

$$f(x) = x^3$$

If you made a table of values and graphed, it would appear as:



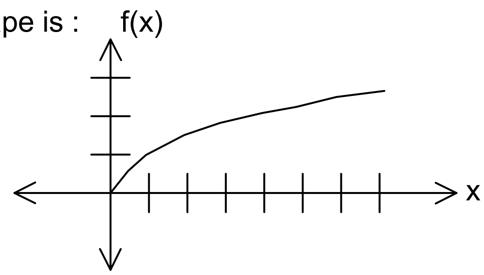
(This kind of looks like a dog's hind leg.)

4. The Square Root Function

Finally, the square root function:

$$f(x) = \sqrt{x}$$

It's basic shape is :



So, these are the four basic shapes we will work with in order to learn how adjustments to the functions of each basic shape will shift, flip, stretch, and compress the basic shape associated with each function.

C. Our Master Example:

We will use the quadratic function and its basic shape with which to learn.

$$f(x) = \pm a(x - h)^2 + k$$

Shifts the basic shape up or down

Shifts the basic shape right or left, the opposite of the way it appears (Note: this number is "trapped" with the x.)

Shape:
- If a = 1, will open the same as the basic shape
- If a > 1, will be stretched vertically (narrower)
- If 0 < a < 1, will be compressed vertically (broader)

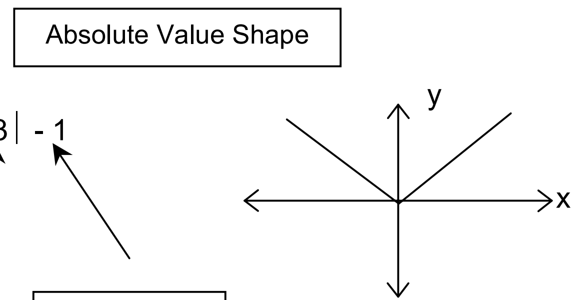
Alignment :
- +, will open same as basic shape (upward for our example)
- -, will be the flip of the basic shape (downward for our example)

(You may wish to note that my use of "a" is really the absolute value of a, $|a|$.)

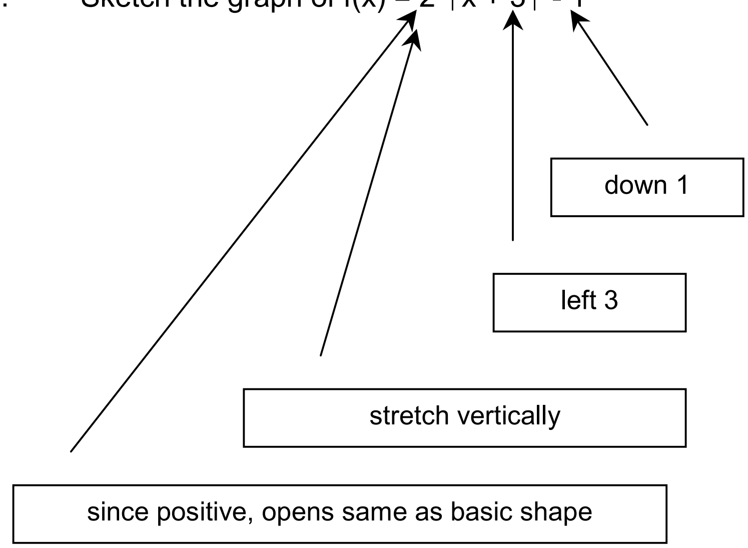
D. Examples:

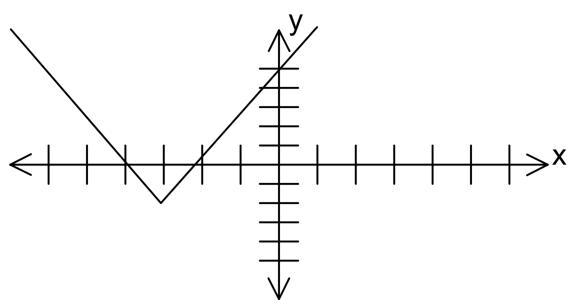
We can now use the Master Example to analyze given functions and predict what their graphs will look like.

The first step, in each example, is to determine which of the four basic shapes we are working with.



1. Sketch the graph of $f(x) = 2\,|x + 3| - 1$





(NOTE: We're shifting the vertex point and the rest of the figure is moving with it, rigid like a wire coat hanger.)

If we were given $f(x) = -\frac{1}{2}(x-2)^3 - 3$, we would move the cubic curve's basic shape: