

Rationalizing the Denominator

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A. What It Means to Rationalize the Denominator

In order that all of us doing math can compare answers, we agree upon a common conversation, or set of rules, concerning the form of the answers.

For instance, we could easily agree that we would not leave an answer in the form of $3 + 4$, but would write 7 instead.

When the topic switches to that of radicals, those doing math have agreed that a RADICAL IN SIMPLE FORM will not (among other things) have a radical in the denominator of a fraction. We will all change the form so there is no radical in the denominator.

Now a radical in the denominator will not be something as simple as $\sqrt{4}$. Instead, it will have a radicand which will not come out from under the radical sign like $\sqrt{3}$.

Since $\sqrt{3}$ is an irrational number, and we need to make it NOT irrational, the process of changing its form so it is no longer irrational is called RATIONALIZING THE DENOMINATOR.

B. There are 3 Cases of Rationalizing the Denominator

1. **Case I:** There is ONE TERM in the denominator and it is a SQUARE ROOT.

Example:
$$\frac{7}{\sqrt{3}}$$

Procedure: Multiply top and bottom by the same radical.

$$\frac{7}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

Look at what is happening here!
 $(\sqrt{3})^2$
Since squaring is the opposite of taking the square root, they cancel each other, leaving the 3.

2. **Case II:** There is ONE TERM in the denominator, however, THE INDEX IS GREATER THAN TWO. It might be a cube root or a fourth root.

Example:

$$\frac{5x}{\sqrt[3]{3x^2}}$$

For the first part of this discussion, we will ignore the top and concentrate on the denominator.

Procedure:

Multiply top and bottom by whatever works in order to create a perfect cube in the denominator.

$$\frac{5x}{\sqrt[3]{3x^2}} \cdot \frac{\sqrt[3]{\quad}}{\sqrt[3]{\quad}}$$

We need to multiply the bottom by something that will make the result a cube...

$$\sqrt[3]{\quad}$$

So we can take the cube root of it here...

$$\quad$$

Which would cause the radical to be gone down here.

Well...

$$\frac{5x}{\sqrt[3]{3x^2}} \cdot \frac{\sqrt[3]{\quad}}{\sqrt[3]{9x}}$$

$$\sqrt[3]{27x^3}$$

It turns out, that we need to multiply the bottom by a 9 so that we get a 27 (which is a cube).

But we also need to multiply it by an x so that we get x^3 (which is also a cube).

$$3x$$

So there, we've gotten rid of the radical in the denominator. We've, therefore, rationalized the denominator. Now we'll look at the entire problem with the numerator included this time.

Remember, whatever we do to the bottom, we must also do to the top.

Original Problem:

$$\frac{5x}{\sqrt[3]{3x^2}}$$

$$\frac{5x}{\sqrt[3]{3x^2}}$$



$$\frac{\sqrt[3]{9x}}{\sqrt[3]{9x}}$$

Here is our adjustment --- what we needed to multiply top and bottom by so the bottom would become a perfect square.

$$\frac{5x^3\sqrt[3]{9x}}{\sqrt[3]{27x^3}}$$

And here we see the result of our multiplication on the bottom. We've created something of which we can take the cube root.

$$\frac{5x^3\sqrt[3]{9x}}{3x}$$

Notice we can cancel the Xs since they are factors and represent the same number.

$$\frac{5\sqrt[3]{9x}}{3}$$

Answer

3. **Case III:** There are TWO TERMS in the denominator.

Example:

$$\frac{\sqrt{3}}{4 - \sqrt{5}}$$

Procedure: We will multiply both top and bottom by the conjugate. The conjugate is the same two terms but with a different sign between them.

Since the conjugate for this denominator is $4 + \sqrt{5}$, we will multiply top and bottom by that number.

$$\frac{\sqrt{3}}{4 - \sqrt{5}} \cdot \frac{4 + \sqrt{5}}{4 + \sqrt{5}}$$

$$\frac{\sqrt{3}(4 + \sqrt{5})}{16 - 5}$$

$$\frac{\sqrt{3}(4 + \sqrt{5})}{11}$$

Note how our use of the conjugate on the bottom is recreating the special product of *The Product of Conjugate Binomials*, as in:

$$(x + 3)(x - 3)$$

Which will always result in squaring everything to get $x^2 - 9$. And, squaring everything would get rid of the term with the square root.

You might need to review the product of two conjugate binomials if you can't see how we got the denominator.

Note: We could distribute the square root of three in the numerator, but there doesn't seem to be any advantage in doing so. Also, we don't have any factors we can cancel, so this is the answer.

REVIEW OF THE PRODUCT OF THE CONJUGATE BINOMIALS...

$$(X + 3)(X - 3)$$

is the product of the **conjugate binomials** because we see the same two terms with different signs between them.

This could also be called "**The product of the sum of two terms and the difference of the two terms.**"

This, then, would be *the sum of X and 3 times the difference of X and 3.*

The product will always be:

The Square of the First Term Minus the Square of the Second Term.

In this example,

$$X^2 - 9$$

will be our answer.