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## Factoring Quadratic Trinomials

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A trinomial is a mathematical expression with 3 terms. A quadratic trinomial is a trinomial whose highest exponent is two. The general form of a quadratic trinomial is  $ax^2 + bx + c$ , where  $a$  is the leading coefficient (number in front of the variable with highest degree) and  $c$  is the constant (number with no variable).

### I. Types of Quadratic Trinomials

1. Quadratic trinomials with a leading coefficient of one.

**Example:**  $x^2 - 12x + 27$   
 $a = 1$     $b = -12$     $c = 27$

2. Quadratic trinomials with a leading coefficient other than one.

**Example:**  $2x^2 + 17x + 26$   
 $a = 2$     $b = 17$     $c = 26$

### II. Factoring Quadratic Trinomials with Leading Coefficient of One (5 steps)

1. List all possible factors of the constant.
2. Determine which factors will add together to give the middle coefficient.  
(\* see Sect IV Hints) Note: If no factors can be found, a different form of factoring must be used.
3. Write two sets of parentheses with x's on the left inside each set.
4. Place the factors inside the parentheses after the x's.
5. Check your answer using the FOIL method and compare to the original trinomial.

**Example:**  $x^2 - 12x + 27$

Step 1) Factors of constant.

1, 27	-1, -27
3, 9	-3, -9

Step 2) Sum of factors equals middle term.

$1 + 27 = 28$	$-1 - 27 = -28$
$3 + 9 = 12$	$-3 - 9 = -12$

Step 3) Set up Parentheses.

$$(x \quad)(x \quad)$$

Step 4) Place correct factors.

$$(x - 3)(x - 9)$$

Step 5) FOIL and Compare.

$$(x - 3)(x - 9)$$

$$x^2 - 3x - 9x + 27$$

$$x^2 - 12x + 27$$

$$\text{original: } x^2 - 12x + 27$$

### **III. Factoring Quadratic Trinomials with Leading Coefficient Other Than One**

1. Multiply the leading coefficient and the constant together.
2. List all possible factors of the result from step one.
3. Determine which factors will add together to give the middle coefficient.  
(\* see Sect IV Hints) Note: If no factors can be found, a different form of factoring must be used.
4. Write the middle coefficient as the sum of the factors using the results from step three and rewrite the polynomial.
5. Group the first two terms and the last two terms together.
6. Factor the Greatest Common Factor (GCF) from each group.
7. Factor the GCF again.
8. Check your answer using the FOIL method and compare to the original trinomial.

**Example:**  $2x^2 + 17x + 26$

Step 1) Multiply  $a$  &  $c$ .

$$2 * 26 = 52$$

Step 2) Factors of  $a * c$ .

$$1, 52 \quad -1, -52$$

$$2, 26 \quad -2, -26$$

$$4, 13 \quad -4, -13$$

Step 3) Sum of the factors equals middle term.

$$1 + 52 = 53 \quad -1 - 52 = -53$$

$$2 + 26 = 28 \quad -2 - 26 = -28$$

$$4 + 13 = 17 \quad -4 - 13 = -17$$

Step 4) Rewrite middle term as Sum of the factors.

$$17x = 4x + 13x$$

Now polynomial looks like  $2x^2 + 4x + 13x + 26$

Step 5) Group Terms.

$$(2x^2 + 4x) + (13x + 26)$$

Step 6) Factor GCF.

$$2x(x + 2) + 13(x + 2)$$

Step 7) Factor GCF again.

$$(x + 2)(2x + 13)$$

Step 8) FOIL and Compare to original trinomial.

$$(x + 2)(2x + 13)$$

$$2x^2 + 13x + 4x + 26$$

$$2x^2 + 17x + 26 \quad \text{original: } 2x^2 + 17x + 26$$

#### IV. Hints on Factoring Quadratic Trinomials

1. If both signs in the trinomial are positive, then both signs in the factored form will be positive.

$$x^2 + 6x + 8$$

positive

$$(x + 2) (x + 4)$$

2. If the first sign is negative and the second sign is positive, then both signs in the factored form will be negative.

$$x^2 - 16x + 39$$

negative positive

$$(x - 13) (x - 3)$$

3. If both signs are negative, then, in factored form, the larger factor will be negative, and the smaller factor will be positive.

$$x^2 - 8x - 33$$

negative positive

$$(x - 11) (x + 3)$$

4. If the first sign is positive and the second sign is negative, then, in factored form, the larger factor will be positive, and the smaller factor will be negative.

$$x^2 + 2x - 35$$

positive negative

$$(x + 7) (x - 5)$$